Homework 8

Due date: Homework is due on Friday, December 5, at 10:00. Homework should be turned into the box for your section (outside of Room 253 Sloan).

1. Following the example developed in class, develop a likelihood ratio test of $H_0 : p = .6$ versus $H_1 : p = .7$ based on observing a binomial random variable with 10 trials.

2. Suppose that the lifetimes of batteries are exponentially distributed with densities

$$f(t|\lambda) == \begin{cases} \frac{1}{\lambda} e^{-t/\lambda} & t \geq 0 \\ 0 & t < 0. \end{cases},$$

   - Calculate the mean lifetime of a battery.
   - Let $T_1, T_2, \ldots, T_n$ be a random sample from $f(t|\lambda)$, derive a likelihood ratio test of $H_0 : \lambda = 1$ versus $H_1 : \lambda = 1.5$, and show that the rejection is of the form $\{\bar{T} \geq c\}$.

3. Moore and McCabe, Chapter 10. Problem 10.4.

4. Old Faithful Geyser in Yellowstone National Park derives its name and its considerable fame from the regularity (and beauty) of its eruptions. As they do with most geysers in the park, rangers post the predicted times of eruptions on signs nearby, and people gather beforehand to witness the show. R.A. Hutchinson, a park geologist, collected measurements of the eruptions (`duration`, in minutes) and the subsequent intervals before the next eruption (`time_between`, in minutes) over an 8-day period (August 1 to August 8, 1978). The data are available on the class webpage (next to the electronic copy of this problem set)

   `http://www.acm.caltech.edu/~emmanuel/math2a/hw.shtml`

   (a) You have been hired by the park to predict the interval between eruptions from the duration of the previous one. Use the data to obtain a method for this prediction problem.

   (b) Suppose that you have just observed an eruption of duration d. Explain how to construct a 95% confidence interval for the **mean value** of the interval until the next eruption.

   (c) Suppose that you have just observed an eruption of duration d. Explain how to construct a 95% confidence interval for the interval until the next eruption; that is an interval I computed from the data so that there is a 95% chance that the time interval until the next eruption will fall in I.

   (d) Give numerical answers to (b) and (c) in the case $d = 4.0$.