Homework 6

Due date: Homework is due on Monday, November 17 at 12:00 Noon. Homework should be turned into the box for your section (outside of Room 253 Sloan).

1. Reading assignment: sections 6.3 and 6.4 in McCabbe.
2. A coin is tossed independently 10 times to test the hypothesis that the probability of heads is 1/2 versus the alternative that the probability is not 1/2. The test rejects if either 0 or 10 heads are observed. What is the significance level of this test? If in fact, the probability of heads is .1, what is the power of this test?
3. This problem introduces the concept of a *one-sided confidence interval*. Using the central limit theorem, how should the constant c^* be chosen so that the interval $(-\infty, \bar{X} + c^* se(\bar{X}))$ is a 90% confidence interval for the population mean μ—i.e., so that $P(\mu \leq \bar{X} + c^* se(\bar{X})) = .9$? This is called a one-sided confidence interval. How should c^* be chosen so that the interval $(\bar{X} - c^* se(\bar{X}), \infty)$ is a 95% one-sided confidence interval
4. Moore and McCabe, Chapter 7. Problem 7.40
5. Moore and McCabe, Chapter 7. Problem 7.64
6. Moore and McCabe, Chapter 7. Problem 7.70
7. In surveys, it is difficult to obtain accurate answers to sensitive questions such as "Have you ever used heroin?" or "Have you ever cheated on an exam?" Warner (1965) introduced the method of randomized response to deal with such situations. A respondent spins an arrow on a wheel or draws a ball from an urn containing balls of two colors to determine which of two statements to respond to: (1) "I have characteristic A," or (2) "I do not have characteristic A." The interviewer does not know which statement is being responded to but merely records a yes or a no. The hope is that an interviewee is more likely to answer truthfully if he or she realizes that the interviewer does not know which statement is being responded to. Let R be the proportion of a sample answering Yes. Let p be the probability that statement 1 is responded to (p is known from the structure of the randomizing device), and let q be the proportion of the population that has characteristic A. Let r be the probability that a respondent answers Yes.
   (a) Show that $r = (2p - 1)q + (1 - p)$.
   (b) If r were known, how could q be determined?
   (c) Show that R is unbiased; that is, $E(R) = r$, and propose an estimate, Q, for q. Show that the estimate is unbiased $(E(Q) = q)$.
   (d) Ignoring the finite population correction, show that
   $$\text{Var}(R) = \frac{r(1-r)}{n}$$
   where n is the sample size.
   (e) Find an expression for $\text{Var}(Q)$.