Homework 4

Due date: Homework is due on Monday, October 27 at 12:00 Noon. Homework should be turned into the box for your section (outside of Room 253 Sloan).

1. **Blood testing.** Suppose that a large number, n, of blood samples are to be screened for a relatively rare disease. If each sample is assayed individually, n tests will be required. On the other hand, it is possible, assuming that the disease is rare, that some savings can be achieved through some pooling procedure. The purpose of this exercise is examine some common pooling procedures.

   Consider the following scheme for group testing. The original lot of samples is divided into two groups and each of the subgroups is tested as a whole. If either subgroup tests positive, it is divided in two and the procedure is repeated. If any of the groups thus obtained tests positive, test every member of that group. (We will assume that the test method is sensitive enough; a group tests positive if and only if at least one person is positive in that group.)

   (a) Find the expected number of tests performed.

   (b) Compare it to the number of tests performed with no groupings. For which value of p is this group testing scheme inferior to testing every individual?

   (c) Consider now the following scheme. The n samples are first grouped into m groups of k samples each, or $n = mk$. Each group is then tested; if a group tests positively, each individual in the group is tested. Find the expected number of tests performed.

2. Pitman, Chapter 3. Exercise 3.2.16.

3. Pitman, Chapter 3. Review exercise 22.

4. Pitman, Chapter 3. Problem 3.3.20.

5. I have 10 cameras. Each time I go on a vacation, I pick up one of the cameras at random and take it with me. If during a particular 10-year period, I went on vacation 20 times, what is the expected number of cameras that were never used during this period?

6. A stick of unit length is broken randomly in two places. What is the average length of the middle piece? (You will assume that the locations of the two break points are independent uniform random variables U_1 and U_2.)

7. **Bonus problem.** (This is a challenging problem!) The King to test a candidate for the position of wise man, offers him a chance to marry the young lady in the court with the largest dowry. The amount of dowries are written on slips of paper and mixed. A slip is drawn at random and the wise man must decide whether that is the largest dowry or not. If he decides it is, he gets the lady and her dowry if he is correct; otherwise he gets nothing. If he decides against the amount written on the first slip, he must choose or refuse the next slip and so on until he chooses one or else the slips are exhausted. In all, 100 attractive young ladies participate with each a different dowry. How should the wise man make his decision?