Solutions to Homework 1

1.4.6

1. If the first card is a spade, then the second card is a spade with probability $\frac{12}{51}$.
2. If the first card is a club, then the second card is a spade with probability $\frac{13}{51}$.
3. Since both of the above cases are equally possible, the probability that the second card is a spade is $\frac{1}{2}\frac{12}{51} + \frac{1}{2}\frac{13}{51} = \frac{25}{102}$.

1.4.8

The probability that the top side is black is $50\%\frac{1}{2} + 20\% = 45\%$. The probability that the top is black and the other side is white is $50\%\frac{1}{2}$. Notice though there are 50% black-white cards, the probability that the top is black and the bottom is white is one half of 50%. So by the general formula of the conditional probability, the chance that the other side is white, given the top is black is $\frac{50\%\frac{1}{2}}{45\%} = \frac{5}{9}$

Chapter 1 Review 4

1.
   - The probability that the picked ball is red is $\frac{1}{2}\frac{2}{5} + \frac{1}{2}\frac{8}{20} = \frac{2}{5}$.
   - The probability that the picked ball is red and from box 1 is $\frac{1}{2}\frac{2}{5} = \frac{1}{5}$.
   - The probability that the picked ball is red and from box 2 is $\frac{1}{2}\frac{8}{20} = \frac{1}{5}$. So the color of the ball is independent of which box is chosen.
2.
   - The probability that the picked ball is red is $\frac{1}{2}\frac{2}{5} + \frac{1}{2}\frac{8}{18} = \frac{19}{45}$.
   - The probability that the picked ball is red and from box 1 is $\frac{1}{2}\frac{2}{5} = \frac{1}{5}$.
   - The probability that the picked ball is red and from box 2 is $\frac{1}{2}\frac{8}{18} = \frac{2}{9}$. So the color of the ball depends on which box is chosen in this case.

Chapter 1 Review 10

(a) The probability that two people 's blood is of the same type is $0.42^2 + 0.1^2 + 0.04^2 + 0.44^2 = 0$. The probability that two people's blood is of the different type is $1 - 0.3816 = 0.6184$.

(b) $P(1) = 0.42^4 + 0.1^4 + 0.04^4 + 0.44^4 = 0.0687$ and $P(4) = 0.42 \times 0.1 \times 0.04 \times 0.44 \times 4! = 0.0177$.

To calculate $P(2)$, we observe that if Type A and Type O appear in the four people, there are three possible combinations: "AAOO", "AOOO" and "AAAO". The probability that "AAOO" appears is $0.42^2 \times 0.44^2 \binom{4}{2}$.

The probability that "AOOO" appears is $0.42 \times 0.44^3 \binom{4}{1}$. Writing the proportion of type j as a_j, we have

$$P(2) = \binom{4}{2} \sum_{i<j} a_j^2 \times a_i^2 + \binom{4}{1} \sum_{i \neq j} a_j \times a_i^3$$

We compute

$$\binom{4}{2} \sum_{i<j} a_j^2 \times a_i^2 = 3((\sum_j a_j^2)^2 - \sum_j a_j^4) = 0.2308$$

and

$$\binom{4}{1} \sum_{i \neq j} a_j \times a_i^3 = 4(\sum a_j \times \sum a_j^3 - \sum a_j^4) = 4 \cdot (0.1603 - 0.0687) = 0.3664$$

Therefore, $P(2) = 0.2308 + 0.3664 = 0.5972$. In addition, $P(3) = 1 - P(1) - P(2) - P(4) = 0.3164$.

Chapter 1 Review 12

(a) Write A_n to be the event that the dice show n different faces, supposing n ordinary dice are rolled and P_n to be the chance that A_n takes place. Write B_j to be the event that j-th dice show a different face from all k-th rolls, $k = 1, 2, \cdots, j-1$.

Since a dice has only six faces, $P(B_n) = 0$ when $n > 6$. It is clear that $P_1 = 1$, $P_2 = \frac{5}{6}$. We claim that $P_k = \prod_{j=0}^{k-1} \frac{6-j}{6}$. Observe that $P(A_n) = P(B_n|A_{n-1})P(A_{n-1})$ and $P(B_n|A_{n-1}) = \frac{6-(n-1)}{6}$.

(b) the event that at least one number appears more than once is the complement of A_n. So its probability is $1 - P(A_n) = 1 - \prod_{j=0}^{n-1} \frac{6-j}{6}$

Candes 4

(a) If the stock is at its original price after two days, then the price must move up and down in each day. So the probability is $2p(1-p)$.

(b) The price must move up twice and move down once in the three days. So the probability is $3p^2(1-p)$.

(c) Write the event that the price moves up one unit after 3 days as A, and the event that the price moves up in the first day as B. The by Bayes' formula, $P(B|A) = \frac{P(AB)}{P(A)}$. Note

that $P(A) = 3p^2(1-p)$ by part (b) and $P(AB) = 2p^2(1-p)$. We conclude that $P(B|A) = \frac{2}{3}$.

Candes 5

(a) $2 \times (\frac{1}{2})^5 = \frac{1}{16}$.
(b) $(\frac{1}{2})^5$
(c) $(\frac{1}{2})^5 \binom{5}{3} = \frac{5}{16}$
(d) $(\frac{1}{2})^2 = \frac{1}{4}$
(e) $1 - (\frac{1}{2})^5$