Solution to the dowry problem

Since we do not know anything about the distribution of the dowries, it seems that a reasonable strategy is to wait until a certain number k of daughters have been presented, and then pick the highest dowry thereafter.

Let D be the position of the highest dowry, e.g. $D = 1$ if the first lady is that with the highest dowry, $D = 2$ if the first lady is that with the highest dowry. We express

$$P(\text{Win}) = \sum_{i=1}^{100} P(\text{Win}|D = i)P(D = i).$$

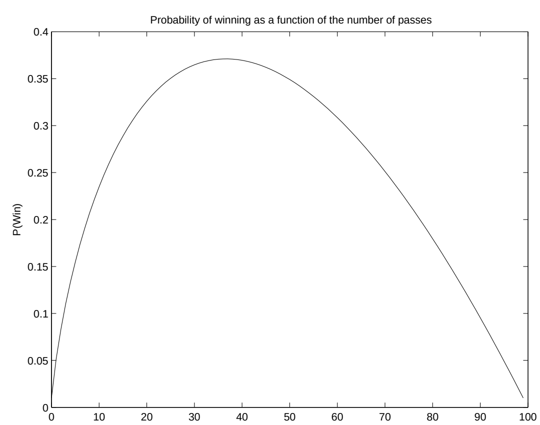
1. $P(D = i) = 1/100$.

2. The probability that you will choose the largest dowry if it is in the ith position is equal to the probability that the highest of the first $i - 1$ dowries belongs to one of the first k ladies. This equals $k/(i - 1)$. If the highest dowry of the first $i - 1$ were not in the first k ladies you would choose it before getting to the largest dowry, thus losing the game.

The probability of winning is then given by

$$\pi_k = \frac{1}{100} \sum_{i=k+1}^{100} \frac{k}{i - 1}$$

Now you want to choose k such that this probability is maximum. Numerical calculations show that $k = 37$ as suggested by the plot below. The probability of winning is about 37.10%.



Let n be the number of dowries. Note that

$$1 + 1/2 + 1/3 + \ldots + 1/m \sim \log(m)$$

and therefore

$$\pi_k \approx \frac{k}{n}(\log(n) - \log(k)) = -\frac{k}{n}\log(k/n)$$

Hence, we want to maximize $-x \log x$ and the maximum is at $1/e$ for which the value is 1/e. In general the answer is going to pass on n/e ladies, where n is the total number of ladies. Because the answer must be an exact integer, check the few integers just above the optimal value.