Homework 4

Due date: Wednesday, November 30.

1. *The Waiting Time Paradox.* Buses arrive at a bus stop at random times according to a Poisson process with intensity λ. I arrive at time t and ask how much time $E(W)$ on average I will have to wait for the next bus. There are two contradictory arguments.

   (a) "The lack of the memory of the Poisson process implies that the distribution of my waiting time should not depend on my arrival time. In this case $E(W) = 1/\lambda$."

   (b) "The time of my arrival is chosen at random in the interval between two consecutive buses, and for reasons of symmetry, my expected waiting time should be half the time the expected time between two consecutive buses, that is $E(W) = 1/(2\lambda)$."

   Which answer is correct? Carefully explain why the other answer is incorrect.

2. Suppose that stars are distributed in space according to a Poisson process of intensity λ. Fix a star *alpha* and let R be the distance to its nearest neighbor. Show that R has the probability density function
$$f_R(x) = 4\lambda\pi x^2 \, e^{-\frac{4\lambda\pi x^3}{3}}, \qquad x > 0.$$

3. Ross (8th edition), Chapter 8, exercise 10.

4. Ross (8th edition), Chapter 8, exercise 18.

5. Suppose three service stations are arranged in tandem so that departures from one form the arrivals for the next. The arrivals to the first station are a Poisson process with rate $\lambda = 10$ per hour. Each station has a single server, and the three service rates are $\mu_1 = 12$ per hour, $\mu_2 = 20$ per hour, and $\mu_3 = 15$ per hour.

   (a) In-process storage is being planned for station 3. What capacity C_3 must be provided if in the long run, the probability of exceeding C_3 is to be less than or equal to 1 percent? That is, what is the smallest number for which $\lim_{t\to\infty} P(X_3(t) > c) < 0.01$?

   (b) Suppose that at $t = 0$, the queues are empty. Simulate the process $(X_1(t), X_2(t), X_3(t))$ for various values of t that you will choose. Do you have a sense of how long it takes to converge to the steady state?