Homework 3

Due date: Wednesday, November 9

1. Let $X(t)$ with $t \in \mathbf{R}$ be a processed defined by

$$X(t) = A\cos\omega t + B\sin\omega t$$

where A and B are i.i.d., zero-mean random variables.

   (a) Show that $X(t)$ is weakly stationary

   (b) Show that $X(t)$ is not strongly stationary.

2. Let $X(t)$ and $Y(t)$ be independent and weakly stationary random processes with zero mean and identical covariance function. Let $Z(t)$ be defined by

$$Z(t) = aX(t) + bY(t)$$

(a, b are not random).

   (a) Determine whether $Z(t)$ is also weakly stationary.

   (b) Find the density of $Z(t)$ if $X(t)$ and $Y(t)$ are also jointly Gaussian random processes.

3. We consider the signal plus noise problem where

$$Y_t = X_t + Z_t,$$

where Y_t are the obeservations, X_t is a random process we wish to recover, and Z_t is white noise with $\sigma = 1$ and independent of X. We suppose that an estimator for X_t uses observations from the following time instants: $I = \{t-p, \ldots, t, \ldots, t+p\}$.

   (a) Find the equation for the optimum filter

   (b) Write the matrix equation for the $2p+1$ coefficients

   (c) Solve for $p = 1$ in the case where X_t is a first-order autoregressive process with autocorrelation $\gamma_X(\tau) = 4 \cdot 2^{-|\tau|}$, $\tau = \ldots, -1, 0, 1, \ldots$.

   (d) Find the mean squared error in part (c).

   (e) *Bonus question.* Perform numerical simulations to show well this works. You may want to simulate an autoregressive Gaussian process, and study the performance of the filter for various values of p.

4. Ross (8th edition), Chapter 5, exercise 26.

5. Ross (8th edition), Chapter 5, exercise 53.

6. Ross (8th edition), Chapter 5, exercise 65.