Simulate the Process

```
function [N1,N2] = runsimulation(lambda,t,mu)
% A function for running one simulation in problem 1(a)
% and get a sample of (N1,N2).
%
% Inputs:
%   lambda intensity of the infection process
%   t time at which we "stop" the process
%   mu mean for the incubation times
% Outputs:
%   N1 number of symptomless infected people at time t
%   N2 number of infected people that show symptoms at time t

% Number of infected people at time t
N = poissrnd(lambda*t);

% Generate the arrival times for the infections (unordered)
T = t*rand(1,N);
```

```
% Generate the incubation times
Tinc = exprnd(mu,1,N);

% times at which symptoms show
Tsymshown = T+Tinc;

% Number of infected people that show symptoms
% at time t
N2 = sum(Tsymshown <= t);

% Number of symptomless infected people at time t
N1 = N-N2;
```

Show Empirical vs. Exact Distribution

```
lambda = 50;
t = 20;

% incubation times
mu = 8;

% number of simulations
B = 10000;
N1 = zeros(1,B);
N2 = zeros(1,B);

% run the simulations
for k=1:B,
  [N1(k),N2(k)] = runsimulation(lambda,t,mu);
end
```

```
[F1,X1] = ecdf(N1);

% Estimate the lambda for N1
lambda1 = lambda*mu*(1-exp(-t/mu));

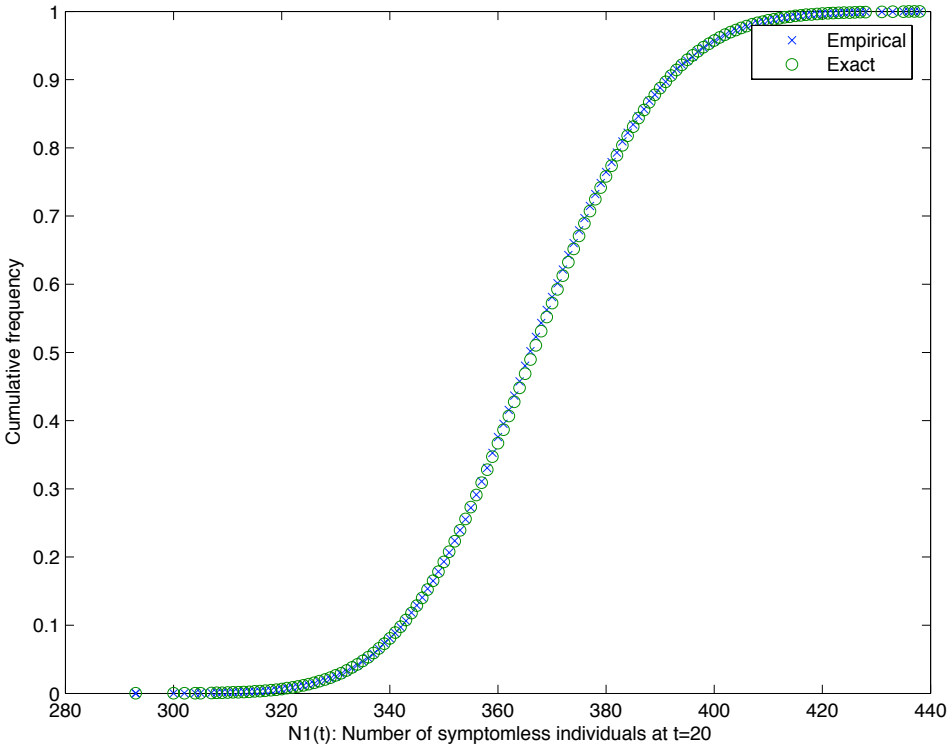
N1cdf = poisscdf(X1,lambda1);

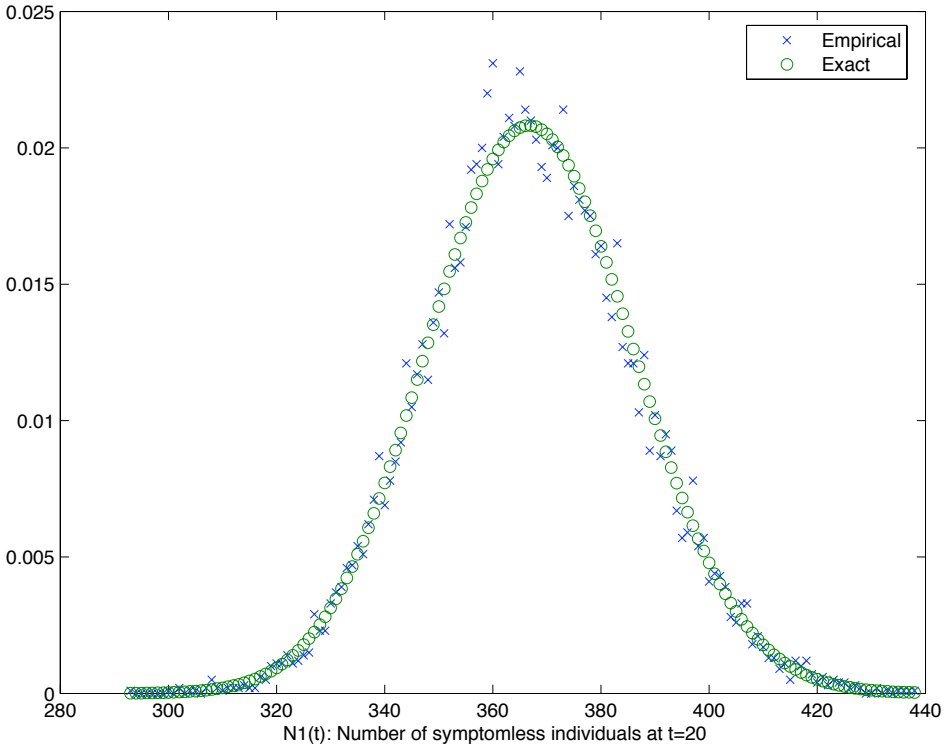
figure(1);
plot(X1,F1,'x',X1,N1cdf,'o');
xlabel('N1(t): Number of symptomless individuals at t=20');
ylabel('Cumulative frequency');
legend('Empirical','Exact');
print -depsc N1empiricalcdf.eps
```

```
X = min(N1):max(N1);
F = histc(N1,min(N1):max(N1));
% normalize
F = F/sum(F);

N1pdf = poisspdf(X,lambda1);

figure(2);
plot(X,F,'x',X,N1pdf,'o');
xlabel('N1(t): Number of symptomless individuals at t=20');
legend('Empirical','Exact');
print -depsc N1empiricalpdf.eps
```


Empirical
Exact
Cumulative frequency
N1(t): Number of symptomless individuals at t=20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
280
300
320
340
360
380
400
420
440


Empirical
Exact
N1(t): Number of symptomless individuals at t=20
0
0.005
0.01
0.015
0.02
0.025
280
300
320
340
360
380
400
420
440

Accuracy of the Prediction

```
p = lambda1/(lambda*t);
q = 1-p;

% The prediction:
N1hat = p/q*N2;

% The prediction errors:
predErrors = N1-N1hat;

% Calculating the mean and standard deviation
% of the prediction errors:
empMeanPredErrors = mean(predErrors)
empStdPredErrors = std(predErrors)

formulaStdPredErrors = sqrt(lambda*p*t/q)
```

```
figure(4);
hist(predErrors,15);
xlabel('Prediction errors');ylabel('Count');
print -deps predErrorsEmpiricalpdf.eps

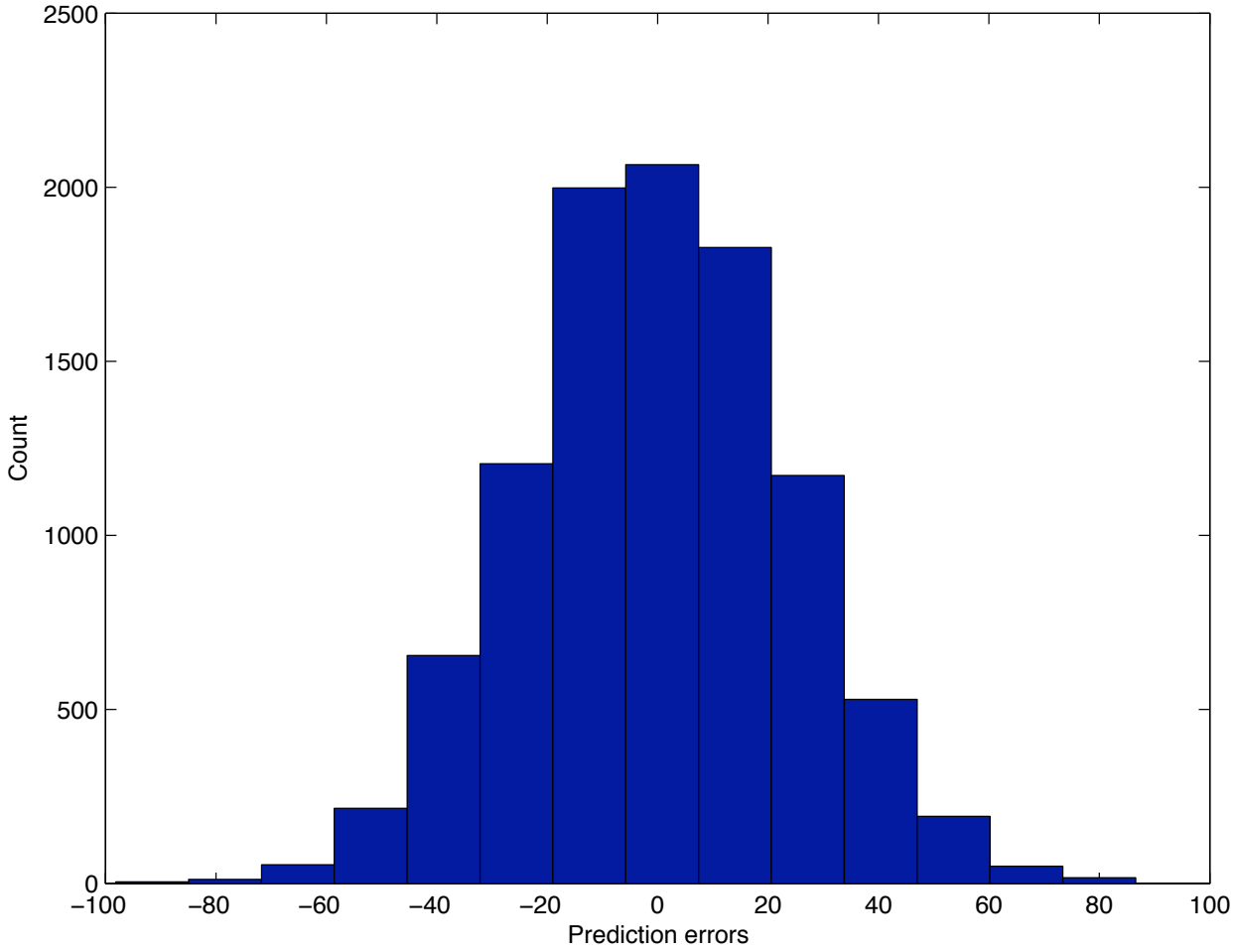
% calculate the percentiles
prctile(predErrors,2.5)
prctile(predErrors,97.5)
```

Formula for std error: 24.0872

Estimated 95% confidence interval for prediciton error $[-\mathbf{48.16}, \mathbf{48.16}]$

Actual percentiles: $-\mathbf{46.9275}$ and $\mathbf{47.5870}$

Excellent agreement


2500
2000
1500
1000
500
0
Count
−100
−80
−60
−40
−20
0
20
40
60
80
100
Prediction errors