Gradient descent

We wish to minimize

$$f(x) = \frac{1}{2}(x_1^2 + Mx_2^2)$$

with $M > 1$. The optimal point is $x^* = (0, 0)$. We use gradient descent with exact line search. The starting point is $x_0 = (M, 1)$.

```
M = 50;
A = diag([1, M])
x0 = [M;1]

NTTOL = 1e-10;     % stop inner iteration if lambda^2/2 < NTTOL
MAXITERS = 500;

% Main loop
histobj=[];x=x0;

for k=1:MAXITERS

   % Evaluate function
   val=x'*A*x/2;

   % Gradient and Hessian: LP part
   g = A*x;
   lambda=-norm(g);  % gradient norm

   % Descent direction
   v = -g;

   histobj=[[histobj],[val;x]]; % Record the function value at current point.

   % Perform exact line search (quadratic function)
   % Value along line .5 (x+tv)' A (x+tv)

   t = -(v'*A*x)/(v'*A*v);
   x = x + t*v;

   % Test is optimum achieved

   if (abs(lambda/2) < NTTOL)   % test if if decrement is smaller than NTTOL
       return;
   end;

x1 = histobj(2,:); x2 = histobj(3,:); plot(x1,x2,x1,x2,'o') axis([-M M -5 5]);
```

```
val = histobj(1,:); figure; semilogy(val);
```

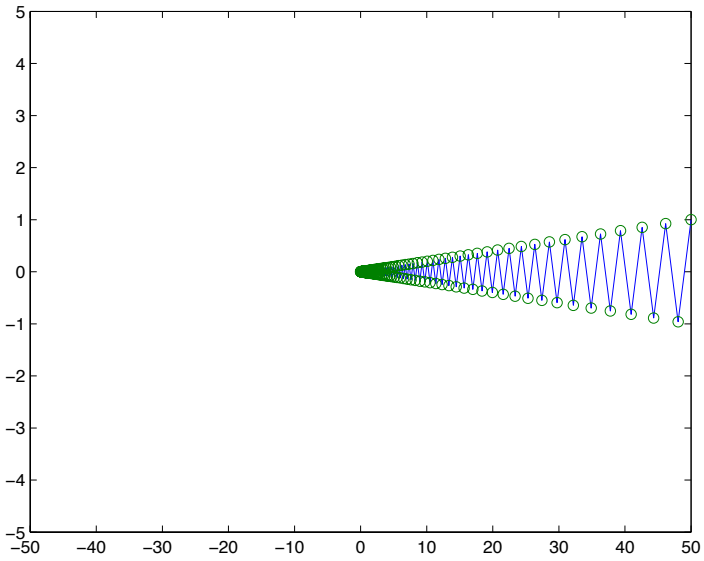


Figure 1: Minimizing sequence $x^{(k)}$.

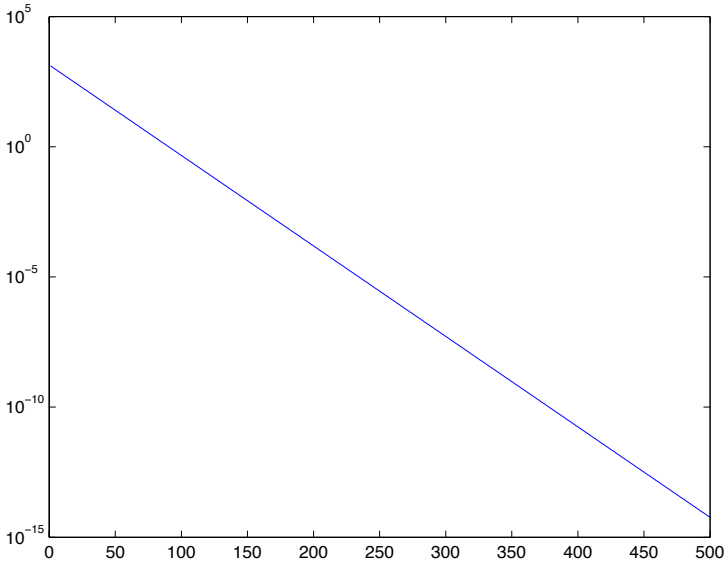


Figure 2: $f(x^{(k)}) - p^*$ as a function of k.