Homework 4

Due date: Friday, November 17

1. A *normal* matrix is a matrix obeying $AA^* = A^*A$.

   (a) Show that a normal triangular matrix must be diagonal.

   (b) Use this result to show that an n by n matrix is normal if and only if it has n orthogonal eigenvectors. (*Hint:* You could first show that a matrix A is normal if and only if its Schur form (i.e. the triangular matrix) is normal.)

2. Threfethen and Bau, Chapter 25, Problem 25.3

3. Threfethen and Bau, Chapter 27, Problem 27.5.

4. This exercise asks you to investigate the speed of convergence of the Rayleigh Quotient Iteration.

   (a) Implement Algorithm 27.3 (in your textbook) in Matlab.

   (b) Form a matrix A as follows

   $$A = S \cdot \Lambda \cdot \Lambda^{-1}, \qquad \Lambda = \mathrm{diag}(1, 2, 6, 30);$$

   Here you will select S at random but with a condition number equal to about 20. You will need to explain how you did it. Please provide your code.

   (c) Pick a few starting points and examine the numerical convergence of the algorithm towards the corresponding eigenvectors and eigenvalues. (Again, you will need to provide your code for this.) Graph your results. You may want to show the error on a log scale as a function of the number of iterations.

   (d) Comment on your findings.