# Homework 1

**Due date: Wednesday, October 4**

1. Threfethen and Bau, Chapter 1, Problem 1.3.
2. Threfethen and Bau, Chapter 3, Problem 3.3.
3. Threfethen and Bau, Chapter 3, Problem 3.4.
4. *Adapted from Demmel.* I plotted below the graph of the function $y = \log(1+x)/x$. Mathematically, $y$ is a smooth function of $x$ near $x = 0$ with $y(0) = 1$. Now for small values of $x$, if we compute $y$ from the formula, we get the plot in Figure (a). Clearly, this formula is unstable near 0. Suppose we use the algorithm

```
d = 1 + x
if d = 1 then
   y = 1
else
   y = log(d)/d-1
end if
```

we get the plot in Figure (b), which is correct near $x = 0$. Explain this phenomenon, proving that the second algorithm must compute an accurate answer in floating point arithmetic. I used IEEE double precision arithmetic, and you may assume that the log function returns an accurate answer from any argument.

For completeness, the computed values at the point $x = -10^{-15} + k10^{-16}$ for $0 \leq k \leq 20$ were:

- naive algorithm:

$$(0.9992, 0.9869, 0.9714, 0.9516, 0.9252, 1.1102, 1.1102, 1.1102, 1.1102, 1.1102, \\ \mathrm{NaN}, 0, 1.1102, 0.7401, 1.1102, 0.8882)$$

- other method:

$$1 + 10^{-15}(0.6661, 0.4441, 0.4441, 0.2220, 0.4441, 0.4441, 0.2220, 0.2220, 0.2220, 0, 0, 0, \\ -0.1110, -0.1110, -0.2220, -0.2220, -0.3331, -0.3331, -0.4441, -0.4441, -0.5551)$$

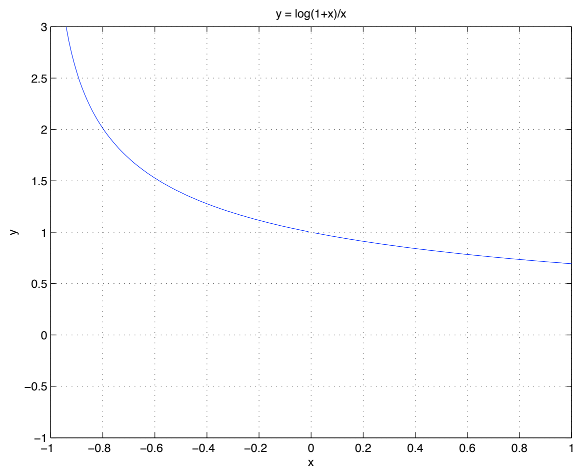
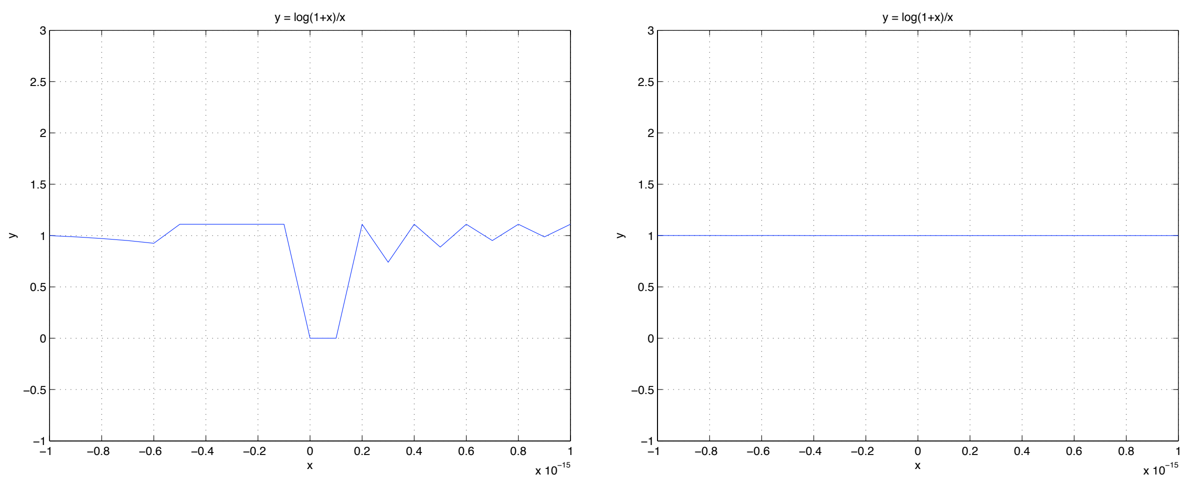



Figure 1: Plot of the function $y = \log(1+x)/x$ in the range $[-1, 1]$.




(a) (b)

Figure 2: Plot of the function $y = \log(1+x)/x$ in the range $[-10^{-15}, 10^{-15}]$.