

Some simple numerics

Emmanuel's implementation of LU

```
function [L,U] = myLU(A)

n = size(A,1);
U = A; L = eye(n);

for k = 1:n-1,
    for j = k+1:n,
        L(j,k) = U(j,k)/U(k,k);
        U(j,k:n) = U(j,k:n) - L(j,k)*U(k,k:n);
    end
end
```

Testing it

```
A = randn(10);
[L, U] = myLU(A);
```

L =

1.0000	0	0	0	0	0	0	0	0	0	0
-4.7591	1.0000	0	0	0	0	0	0	0	0	0
1.0813	-0.2504	1.0000	0	0	0	0	0	0	0	0
0.6829	-0.2090	-14.0008	1.0000	0	0	0	0	0	0	0
-3.7415	0.5142	4.8920	-0.1543	1.0000	0	0	0	0	0	0
6.0090	-0.9774	-7.5775	0.3417	-1.2901	1.0000	0	0	0	0	0
6.0576	-1.0145	-6.2708	0.6249	1.7039	-0.2652	1.0000	0	0	0	0
-1.5823	0.0985	6.4597	-0.4478	0.5602	0.4839	0.7000	1.0000	0	0	0
-1.4429	0.3645	-0.1860	-0.0155	-0.0597	-0.5049	0.4765	0.6951	1.0000	0	0
0.6730	-0.1488	-5.5518	0.3052	-0.6077	-0.1212	-0.4498	-0.8299	2.2234	1.0000	0

U =

0.2920	1.1619	0.5872	-0.2427	-0.4357	-0.4854	0.4913	0.4605	-0.5257	0.0741	0
0	6.6432	1.4838	-1.1834	-1.8363	-1.5321	2.4151	0.4917	0.7312	0.8894	0
0	0	0.1819	-0.5645	-2.0335	0.4829	-1.0651	0.3082	-0.1470	-1.3734	0
0	0	0	-8.4907	-28.3303	8.3366	-15.2626	3.7473	-2.6530	-19.4137	0
0	0	0	0	3.6008	-0.5991	3.1921	0.2370	-1.9216	2.8846	0
0	0	0	0	0	2.3276	0.6393	-1.4071	1.3173	0.1184	0
0	0	0	0	0	0	-3.4886	-1.9551	8.0718	-3.2955	0
0	0	0	0	0	0	0	1.4885	-5.7989	2.6325	0
0	0	0	0	0	0	0	0	-0.9701	-0.1876	0
0.0000	0	0	0.0000	0	0	0	0	0	0.9480	0

```
norm(A - L*U)/norm(A)
```

```
ans =
```

```
5.9505e-16
```

```
norm(A - L*U,'fro')/norm(A,'fro')
```

```
ans =
```

```
4.6219e-16
```

Instability of LU decomposition

```
epsilon = 1e-20;
```

```
A = [epsilon 1; 1 1]
```

```
A =
```

```
0.0000    1.0000  
1.0000    1.0000
```

```
[L, U] = myLU(A);
```

```
tildeA = L*U
```

```
tildeA =
```

```
0.0000    1.0000  
1.0000         0
```

```
b = [1; 0];
```

```
x = A\b
```

```
x =
```

```
-1  
1
```

```
% This is is the correct solution
```

```
x = tildeA\b
```

```
x =
```

```
0  
1
```

```
% This is completely wrong!
```

The difference between $A \setminus b$ and $\text{inv}(A) * b$

```
n = 10000;  
A = randn(n);
```

```
tic, x = A \ b; toc
```

Elapsed time is 21.228614 seconds.

```
tic, invA = inv(A); toc
```

Elapsed time is 60.599312 seconds.