Homework 6

Due date: Friday, March 2

1. The differentiation operator on the space of polynomials of degree less or equal to 3 is represented in the natural ordered basis by the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

   What is the Jordan form of this matrix?

2. If A is an $n \times n$ matrix with characteristic polynomial

$$P(\lambda) = (\lambda - \lambda_1)^{d_1} \dots (\lambda - \lambda_k)^{d_k}$$

   what is the trace of A?

3. Let n be a positive integer, $n \geq 2$, and let N be an $n \times n$ matrix such that $N^n = 0$ but $N^{n-1} \neq 0$. Prove that N has no square root, i.e., that there is no $n \times n$ matrix A such that $A^2 = N$.

4. What's wrong with the following proof? If A is a complex $n \times n$ matrix such that $A^T = -A$, then $A = 0$. Proof: Let J be the Jordan form of A. Since $A^T = -A$, $J^T = -J$. But J is triangular so that $J^T = -J$ implies that every entry of J is zero. Since $J = 0$ and A is similar to J we see that $A = 0$. Give an example of a nonzero matrix such that $A^T = -A$.

5. (a) Find a nonzero matrix N such that $N^3 = 0$.
   (b) If $Nx = \lambda x$, show that λ must be zero.
   (c) Prove that N cannot be symmetric.

6. Franklin Chapter 5, Problem 3, page 140.

7. Franklin Chapter 5, Problem 5, page 140.