Homework 4

Due date: Wedesday, January 14

1. Franklin Chapter 2, Problem 4, page 55.
2. Franklin Chapter 4, Problem 3, page 75.
3. Franklin Chapter 4, Problem 4, page 75.
4. Franklin Chapter 4, Problem 5, page 88.
5. Franklin Chapter 4, Problem 4, page 94.
6. Let B be an $m \times n$ matrix and A be a square invertible matrix ($m \times m$). Prove that AB and B have the same nullspace and the same row space (and the same rank) as B itself. Use this fact to show that the rank of a matrix is an invariant.
7. Let A and B be two square matrices. Prove that the characteristic polynomials of A and AB are identical. (Hint: It may be helpful first to establish the result for $\det A \neq 0$. In the case where $\det A = 0$, think about 'perturbing' the matrix A such that it becomes invertible.)
8. A projection matrix is a matrix which satisfies the identity $P = P^2$.
   (a) If $P = P^T P$ show that P is a projection matrix
   (b) If P is an orthogonal projection matrix, show that $P = P^T$.
   (c) Suppose that P is the projection onto the subspace M and Q is the projection onto the orthogonal complement $M^\perp$. What are $P+Q$ and PQ? Show that $P-Q$ is its own inverse.