Homework 3

Due date: Tuesday, January 30

1. Reading assignment: Franklin, Chapter 3.
2. Franklin Chapter 2, Problem 2, page 50.
3. Franklin Chapter 4, Problem 5, page 80.
4. Franklin Chapter 4, Problem 4, page 83.
5. Let u be a vector in $\mathbb{R}^n$ with coordinates $(u_1, u_2, \ldots, u_n)$ and let P be the orthogonal projection onto the line $\{\lambda u, \lambda \in \mathbb{R}^n\}$.
   (a) Show that this is a linear transformation.
   (b) Find the matrix of P in the canonical basis. We will also call this matrix P. (The canonical basis $e_1, \ldots, e_n$ is the basis where e_i is the vector with a one on the ith position and zeroes everywhere else.)
   (c) What is P^2?
   (d) What is the trace of P? What is the rank of P?
   (e) Repeat (a), (c) and (d) where P is now the orthogonal projection onto r linearly independent vectors. What is P in the case $r = n$?
6. Let A be an arbitrary matrix. Show that A is not similar to $A + I$.
7. Consider any A and a special "plane rotation" M.

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \quad M = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Choose the rotation θ as to annihilate the $(3, 1)$ entry of $M^{-1}AM$.